P6 - Rational Expressions

Division of 2 polynomials and the denominator cannot $= 0$.

Step 1 Factor the numerator

Step 2 Factor the denominator

Step 3 Cross out

$\frac{4}{x-1}$ $\{x \mid x \neq 1\}$

$\frac{1}{(x-3)(x+5)}$ $\{x \mid x \neq 3 \text{ and } x \neq -5\}$

$$\frac{x}{x^2-36}$$

$$\frac{x}{(x+6)(x-6)} \quad \{x \neq 6, x \neq -6\}$$

* Write what makes $x=0$ based on the original equation

P. 75

$$\frac{x^3+x^2}{x+1}$$

$$\frac{x^2(x+1)}{x+1} = x^2$$

$x \neq -1$

$$\frac{x^2+6x+5}{x^2-25}$$

$1x \quad 1x$

add 6 : $5+1$ | $5 \cdot 1 = 5$ (a·c)

$$\frac{(x+5)(x+1)}{(x+5)(x-5)}$$

$$\frac{x+1}{x-5}$$

$x \neq 5 \quad x \neq -5$

$$\frac{x^2+6x+5}{9x^2-81} = \frac{(x+5)(x+1)}{9(x^2-9)}$$

$$\frac{(x+5)(x+1)}{9(x+3)(x-3)}$$

$x \neq -3$

$x \neq 3$

$$\frac{x^3+3x^2}{x+3} = \frac{x^2(x+3)}{x+3} = x^2,\ x \neq -3$$

$$\frac{x^2-1}{x^2+2x+1} = \frac{(x+1)(x-1)}{(x+1)(x+1)} = \frac{x-1}{x+1},\ x \neq -1$$

Multiply Rational Expressions

Follow the 3 steps and find zeros for the denominators

Ex 3, p. 76

$$\frac{x-7}{x-1} \cdot \frac{x^2-1}{3x-21}$$

$$\frac{x-7}{x-1} \cdot \frac{(x+1)(x-1)}{3(x-7)}$$

$$\frac{x+1}{3}, \quad x \neq 1, 7$$

$$\frac{x+3}{x^2-4} \cdot \frac{x^2-x-6}{x^2+6x+9}$$

$$\frac{\cancel{x+3}}{\cancel{(x+2)}(x-2)} \cdot \frac{(x-3)\cancel{(x+2)}}{\cancel{(x+3)}(x+3)}$$

$$\frac{x-3}{(x-2)(x+3)}, \quad x \neq 2, -2, -3$$

p. 82-83,

(2) $x \neq -9$

(4) $x \neq 7, -7$

(6) $x \neq 9, -5$

(8) $\frac{4}{x-2}, x \neq 2$

(10) $\frac{x-4}{3}, x \neq 4$

(12) $\frac{y-5}{y+4}$ $y \neq -4, -1$

Division of rational expressions:

You never ÷ rational expressions.

Take the inverse of the divisor:

* Keep Change Flip *

p. 76

$$\frac{x^2-2x-8}{x^2-9} \div \frac{x-4}{x+3}$$

$$\frac{x^2-2x-8}{x^2-9} \cdot \frac{x+3}{x-4}$$

Finish it

Division

Exclude zeros in the denominator for the original equation AND after you flip it

$$\frac{x^2-2x-8}{x^2-9} \cdot \frac{x+3}{x-4}$$

$$\frac{(x-4)(x+2)}{(x+3)(x-3)} \cdot \frac{x+3}{x-4}$$

$$\frac{x+2}{x-3} \qquad x \neq 3, -3, 4$$

$$\frac{x^2-2x+1}{x^3+x} \div \frac{x^2+x-2}{3x^2+3}$$

Adding/Subtracting Rational Expressions

* You need a common denominator (Just like fractions)

This applies only to addition/subtraction

p. 77 $\frac{5x+1}{x^2-9} - \frac{4x-2}{x^2-9}$ } same denominator

$$\frac{5x+1-(4x-2)}{x^2-9}$$

$$\frac{5x+1-4x+2}{x^2-9} = \frac{x+3}{x^2-9}$$ Now factor

$$\frac{x+3}{(x+3)(x-3)} = \frac{1}{x-3} \quad x \neq 3, -3$$

$$\frac{x}{x+1} - 1\left(\frac{3x+2}{x+1}\right)$$

$$\frac{x-3x-2}{x+1} = \frac{-2x-2}{x+1}$$

$$\frac{-2(x+1)}{x+1} = -2 \quad x \neq -1$$

$$\frac{x}{x+1} - \frac{3x-2}{x+1}$$

$$\frac{x-3x+2}{x+1} = \frac{-2x+2}{x+1}$$

$$= \frac{2(x-1)}{x+1} \qquad x \neq -1$$

$$\frac{x^2-4x}{x^2-x-6} + \frac{4x-4}{x^2-x-6}$$

$$\frac{x^2-4}{(x-3)(x+2)} = \frac{(x+2)(x-2)}{(x+2)(x-3)}$$

$$= \frac{x-2}{x-3}$$

$x \neq 3, -2$

Add/Subtract with different denominators:

P. 78-80 | Find the LCD (least common denominator). Factor if possible

$\frac{1}{4} + \frac{2}{3}$

You multiply each fraction by what's missing to make the denominators = 12

$$\frac{1}{x^2-4} + \frac{3}{x-2}$$ } Make denominators equal

What's missing in denominators?

$$\frac{1}{(x+2)(x-2)} + \frac{3}{x-2} \cdot \frac{x+2}{x+2}$$

$$\frac{1}{(x+2)(x-2)} + \frac{3(x+2)}{(x+2)(x-2)} = \frac{1+3x+6}{(x+2)(x-2)}$$

$$\left(\frac{3x+7}{(x+2)(x-2)}, \{x \mid x \neq 2, -2\}\right)$$

Ex 6, p. 78

$$\frac{x+2}{2x-3} - \frac{4}{x+3}$$

What's missing from denominators?

$$\frac{(x+2)(x+3)}{2x-3(x+3)} - \frac{4(2x-3)}{(x+3)(2x-3)}$$

$$\frac{x^2+5x+6-8x+12}{(2x-3)(x+3)}$$

$$\frac{x^2+5x+6-8x+12}{(2x-3)(x+3)}$$

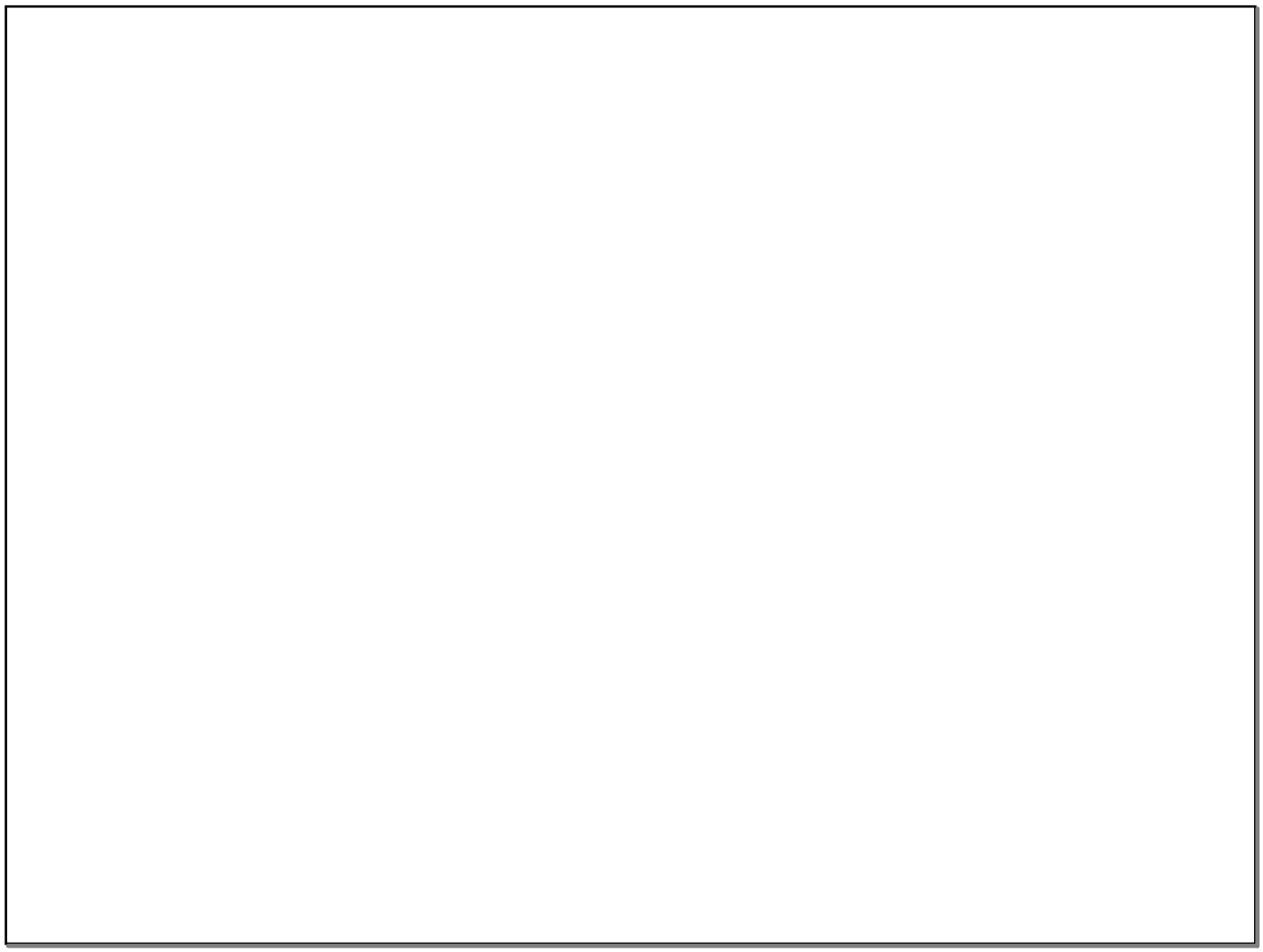
$$\frac{x^2-3x+18}{(2x-3)(x+3)} = \frac{x^2-3x+18}{(2x-3)(x+3)}$$

$$x = -3, \frac{3}{2}$$

$$\frac{7}{5x^2+15x} + \frac{9}{x^2+6x+9}$$

*Identify what is missing in each denominator

February 10, 2012



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$$\frac{x+3}{x^2+x-2} + \frac{2}{x^2-1}$$

Simplifying Complex Rational Expressions

P. 81 – (Ignore method shown in Ex. 9)

* Find the LCD of the numerator and denominator

$$\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$$

x is the LCD of everything so multiply the top and bottom by x

$$\frac{\left(1+\frac{1}{x}\right)x}{\left(1-\frac{1}{x}\right)x}$$

LCD $= x$

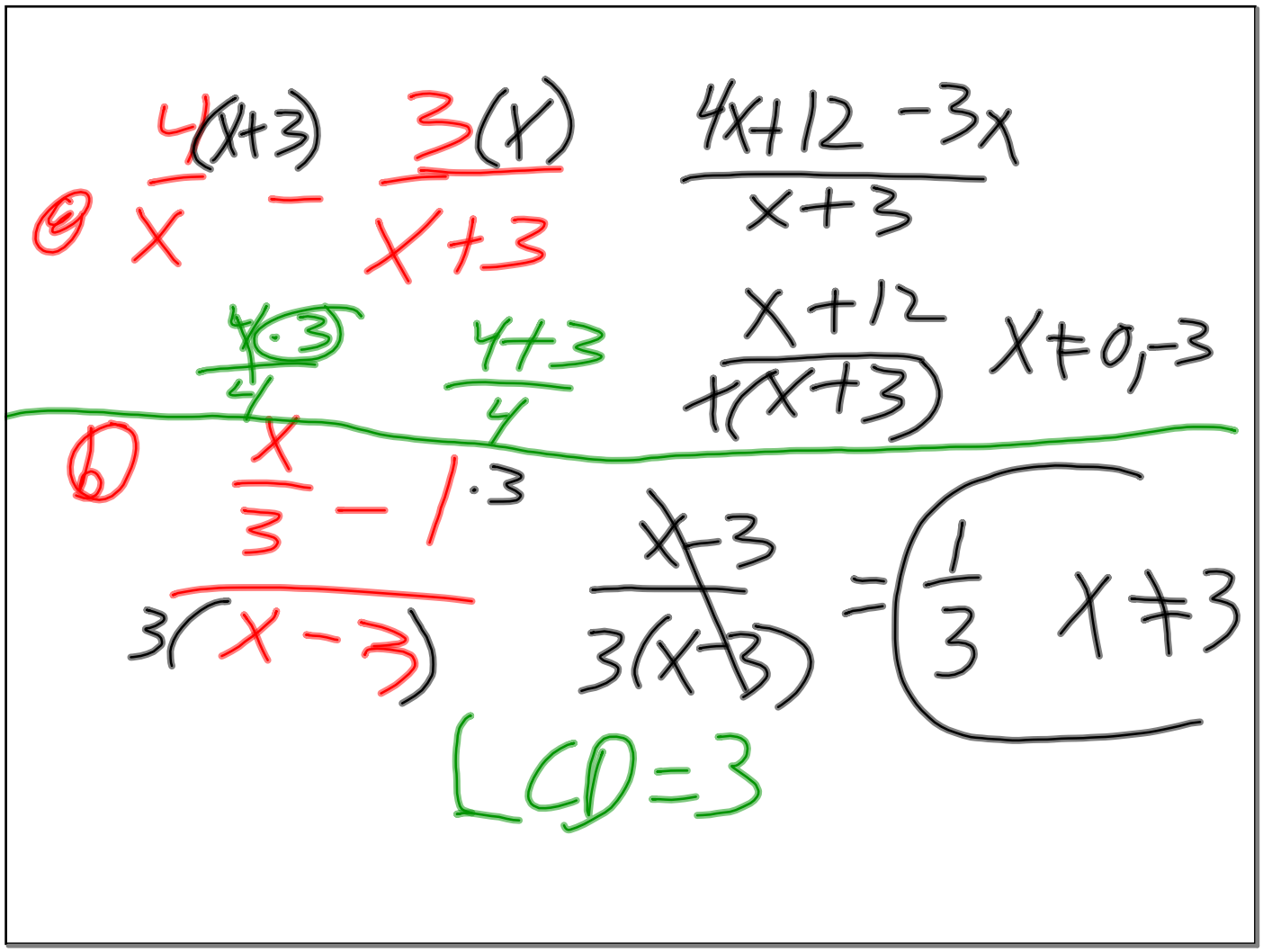
$$\frac{x+1}{x-1} \qquad x \neq 0, 1$$

$$\frac{4\cdot\frac{1}{x}-\frac{3}{2}\cdot 2x}{4\cdot\frac{1}{x}+\frac{3}{4}\cdot x}$$

LCD $= 4x$

$\frac{1}{4}\cdot 4 = 1$ $\quad \frac{3}{4}\cdot 4 = 3$

$$\frac{4-6x}{4+3x} = \frac{2(2-3x)}{4+3x} \qquad x \neq 0, -\frac{4}{3}$$


4(x+3) 3(x) 4x+12 -3x
(a) x − x+3 x+3
4·3 4+3 x+12
4 4 x(x+3) x≠0,-3
(b) x/3 − 1 ·3
3(x-3)
x-3 = 1/3 x≠3
3(x-3)
LCD=3

$$(-4^{1}b^{6})^{-4}$$

$$-4^{-4}b^{-24}$$

$$\frac{1}{-4^{4}b^{24}}$$

$$\frac{1}{256b^{24}}$$

February 10, 2012

$$\frac{\sqrt{11}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{55}}{5}$$

$$(-5x^{\frac{3}{5}})(6x^{\frac{4}{5}})$$

$$-30x^{\frac{3}{5}+\frac{4}{5}}$$

$$-30x^{\frac{7}{5}} = -30x\sqrt[5]{x^2}$$

$$x^{\frac{7}{5}} =$$

$$\frac{\left(-3^{1} y^{\frac{3}{4}}\right)^{4}}{y^{\frac{3}{2}}} = \frac{-3^{4} y^{\frac{3 \cdot 4}{4}}}{y^{\frac{3}{2}}}$$

$$\frac{81 y^{3}}{y^{\frac{3}{2}}} = 81 y^{\frac{3}{1} - \frac{3}{2}}$$

$$81 y^{\frac{6}{2} - \frac{3}{2}} = 81 y^{\frac{3}{2}}$$

$$\frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

LCD of everything is $x(x+h)$. h in the denominator is really $\frac{h}{1}$

$$\frac{\frac{1 \cdot x(x+h)}{x+h} - \frac{1 \cdot x(x+h)}{x}}{h \cdot x(x+h)} = \frac{\frac{x\cancel{(x+h)}}{\cancel{(x+h)}} - \frac{\cancel{x}(x+h)}{\cancel{x}}}{}$$

$$= \frac{x - x + h}{hx(x+h)} = \frac{\cancel{h}}{\cancel{h}x(x+h)} = \frac{1}{x(x+h)}, \; h \neq 0, x \neq 0$$

(65) $\dfrac{X - \dfrac{X}{X+3}}{X+2}$ $\quad LCD = X+3$

$\dfrac{X}{1}$ $\quad \dfrac{X+2}{1}$

$$\frac{\left(X - \frac{X}{X+3}\right)(X+3)}{(X+2)(X+3)} = \frac{X(X+3) - X(X+3)}{(X+2)(X+3)}$$

$$\frac{X^2+3X-X}{(X+3)(X+2)} = \frac{X^2+2X}{(X+3)(X+2)} = \frac{X(X+2)}{(X+2)(X+3)} = \frac{X}{X+3}$$

$X \neq -3, -2$

February 10, 2012