P5 - Factoring Polynomials

1st Step always is to factor a GCF (greatest common factor) if possible:

$$12y^2 - 3y^3 = 3y^2(4 - y)$$

$$70x^8 + 14x^5 = \cancel{7x^5(10x^3+2)}$$

$$14x^5(5x^3+1)$$

$70x^8 + 14x^5$

$14x^5(5x^3 + 1)$

$3x^2 - 9x$

$3x(x-3)$

$3x^2 - 9x = 3x(x-3)$

$x^2(x+3)+5(x+3)$

$x^2\cdot 1 \quad + 5\cdot 1$

$(x+3)(x^2+5)$

Group to get the GCF if there are 4 terms: (p. 63)

$x^3+4x^2+3x+12$

$(x^3+4x^2)+(3x+12)$

① Put parentheses around 1st 2 terms and last 2 terms

$x^2(x+4)+3(x+4)$

② Remove a GCF if possible

$\boxed{(x+4)(x^2+3)}$

$$(x^3+5x^2)(-2x-10)$$

$$x^2\cancel{(x+5)}-2\cancel{(x+5)}$$

$$(x+5)(x^2-2)$$

$$(x^3+6x^2)+(-2x-12)$$

$$x^2(x+6)+-2(x+6)$$

$$(x+6)(x^2-2)$$

Factoring:
1) Always look at your “b” and “c”:
a) If “b” and “c” are positive than the factors of “c” are positive and the operations in the binomial are both addition.
b) If “b” is negative and “c” is positive then the factors of “c” are negative and the operations in the binomial are both subtraction.
c) If “b” is positive or negative and “c” is negative then the factors of “c” are negative and one operation is addition and one is subtraction.
*** a) through c) are extremely helpful ***

2) See if any variable and/or # is a Greatest Common Factor of ALL the terms
3) Simplify into 2 binomials.

Example:

$x^2 + 8x + 15$ You can't factor anything out of each term.
Your "b" and "c" are both positive.

Step 1: Identify the values of a, b and c
$a = 1$ $b = 8$ $c = 15$

Step 2: In separate columns - List "a" twice , next put "+/- = b" and then "a * c"

Step 3: *** Look at "b" and "a *c". "a * c" is the result of multiplication. "b" is the result of addition/subtraction SO:

Find the factors of "a * c" that multiply to 15 and add to 8. This would 3 and 5

Step 4: Put the factors under the two "a"s. Reduce if possible

Step 5: Place the "fractions" into the binomials in the same order.

Check it by using FOIL on your answer. It better equal your original trinomial.

ⓐ $x^2+4x+3 \quad = (\quad + \quad)(\quad + \quad)$

ⓑ $x^2-4x+3 \quad (\quad\quad)(\quad\quad)$

ⓒ₁ x^2-2x-3

ⓒ₂ x^2+2x-3

$x^2 + 4x + 3$

$a = 1$
$b = 4$
$c = 3$

① write "a" twice

$\underset{3}{\underline{1x}}\ \underset{1}{\underline{1x}} = (x+3)(x+1)$

② I need factors that multiply to "a·c" and add to "b"

$3 \cdot 1 = 3$ $\quad 3 + 1 = 4$

$$x^2 + 13x + 40$$

$\cancel{1}x$	$\_\_x$	add to b	a · c multiply
8	5		1 · 40
			2 · 20
		8 + 5	10 · 4
			8 · 5

$(x+8)(x+5)$

$x^2+3x-18$

$a=1$
$b=3$
$c=-18$

$\frac{x}{-3}$ $\frac{x}{6}$

add
$\frac{+\ \text{to}\ 3}{-3}$

$\frac{a \cdot c = -18}{\cancel{-6 \cdot 3}}$
$6 \cdot -3$

$(x-3)(x+6)$

$$8x^2 - 10x - 3$$

$\dfrac{\cancel{8x}}{\cancel{-12}}$ $\dfrac{\cancel{8x}}{\cancel{2}}$ $\dfrac{\text{add to } -10}{-12+2}$ $\dfrac{a \cdot c = -24}{-12 \cdot 2}$

$\dfrac{2x}{-3}$ $\dfrac{4x}{1}$ $\boxed{(2x-3)(4x+1)}$

$$2x^2 - 7xy + 3y^2$$

$$\frac{\cancel{2}\ x}{-2} \quad \frac{2x}{-1} \quad \frac{\text{add to } -7}{-6 + -1} \quad \frac{a \cdot c = 6}{-6 \cdot -1}$$

$$\frac{x}{-3} \quad \frac{2x}{-1}$$

$$(x - 3y)(2x - 1y)$$

P.66 Factoring a trinomial with Two variables

$2x^2 - 7xy + 3y^2$

* Solve as if the 2^{nd} variable is not there

$a=2$ $b=-7$ $c=3$

$2x^2 - 7x + 3$

$\frac{2x}{-6}$ $\frac{2x}{-1}$ $\frac{\text{add } b=-7}{-6+-1}$ $\frac{a \cdot c = 6}{-6 \cdot -1}$

$(x-3)(2x-1)$

* now add in the 2^{nd} variable

$(x-3y)(2x-1y)$

Perfect square trinomials (p. 67-68)

* If the first and last terms are perfect squares then shortcuts apply (You can always still factor the regular way):

$$\underbrace{9x^2 - 121}_{\text{no middle term}} = (3x+11)(3x-11)$$

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$$x^4 - 81 = (x^2+9)(x^2-9)$$

$$= (x^2+9)(x+3)(x-3)$$

Ex 9, P. 68

$x^2 + 6x + 9$

1 and 9 are perfect squares

Look at signs so answer is:

$(+)(+)$

Put in perfect squares

$(1x+3)(1x+3)$

$x^2 - 12x + 36$

Factoring cubic Polynomials:

S O A P

Same signs, opposite signs, always positive

(binomial) (trinomial)

$a^3 + b^3$

$a^3 - b^3$

$$x^3 + 8 = (A+B)(A^2 - AB + B^2)$$

S

$$(x+2)(x^2 - 2x + 4)$$

$$64x^3 - 125 = (A-B)(A^2 + AB + B^2)$$

$$(\underbrace{4x}_{A} - \underbrace{5}_{B})(16x^2 + 20x + 25)$$

$$64x^3 - 125 = (A - B)(A^2 + AB + B^2)$$

$$(4x - 5)(16x^2 + 20x + 25)$$

$$(5x - 1)(25x^2 + 5x + 1)$$

A B

p.71 all 65-72

65 $3x^3-3x$

$3x(x^2-1)$

$3x(x+1)(x-1)$

66 $5x(x+3)(x-3)$

68 $6(x+2)(x-5)$

70 $7(x^2+1)(x+1)(x-1)$

72 $(x-5)(x+5)(x-3)$