

P.3: Radicals and rational exponents

$$a^{\frac{1}{n}}$$

$$9^{\frac{1}{2}}$$

$$8^{\frac{2}{3}}$$

$$\sqrt[3]{9^1} = 3 \quad | \quad \sqrt[3]{8^2} = \sqrt[3]{64} = 4$$

Rational exponents - the power is a fraction:

- ① The numerator becomes the power of the radicand
- ② The denominator becomes the index.

$$8^{\frac{2}{3}} \quad - \text{ solve 2 ways}$$

$$\textcircled{1} \sqrt[3]{8^2} = \sqrt[3]{64} = 4 \quad \text{OR:}$$

$$\textcircled{2} (\sqrt[3]{8})$$

$$= 2$$

$$2^2 = 4$$

Ⓐ Put the base with the index and solve for the root

Ⓑ Add the numerator as the power and solve.

Rational exponent with a
negative power

$$8^{-\frac{1}{3}} = 8^{\frac{1}{3}} = \frac{1}{\sqrt[3]{8}} = \left(\frac{1}{2}\right)$$

* Get rid of a negative power by moving it from the numerator to the denominator or vice-versa.

$$\sqrt[3]{-64} = -4$$

$$\sqrt{-64} = \text{undefined}$$

$$-32^{\frac{1}{5}} = \sqrt[5]{-32} = (-2)$$

$$x^{\frac{5}{3}} = \sqrt[3]{x^5} = x \sqrt[3]{x^2}$$

$$\sqrt[4]{8x^3} \cdot \sqrt[4]{4x^2}$$

$$\sqrt[4]{32x^5} =$$

$$\sqrt[4]{16} \cdot \sqrt[4]{2} \cdot x \sqrt[4]{x}$$

$$(2x \sqrt[4]{2x})$$

Operations with rational Exponents

Multiplication:

$$2^{\frac{1}{2}} \cdot 2^{\frac{3}{4}} = 2^{\frac{1}{2} + \frac{3}{4}} = 2^{\frac{5}{4}}$$

$$\sqrt[4]{2^5} = 2 \sqrt[4]{2}$$

There is a 2
you can pull
out

Final answer

$$x^{\frac{1}{2}} \cdot x^{\frac{3}{4}} =$$

$$5' x^{\frac{1}{2}} \cdot 7' x^{\frac{3}{4}} = 35 x^{\frac{1}{2} + \frac{3}{4}}$$

$$35 x^{\frac{5}{4}} = 35 \sqrt[4]{x^5}$$

$$= 35 x \sqrt{x}$$

$$\frac{2^{\frac{3}{4}}}{2^{\frac{1}{2}}} = 2^{\frac{3}{4} - \frac{1}{2}} = 2^{\frac{1}{4}} = \sqrt[4]{2}$$

$$\frac{X^{\frac{3}{4}}}{X^{\frac{1}{2}}} = X^{\frac{3}{4} - \frac{1}{2}} = X^{\frac{1}{4}} = \sqrt[4]{X}$$

$$\frac{10' x^3}{2' x^{\frac{1}{4}}} = \left(\frac{10'}{2'}\right) \left(x^{3-\frac{1}{4}}\right)$$

$$= 5 x^{\frac{12}{4}-\frac{1}{4}} = 5 x^{\frac{11}{4}} = 5 x^{\frac{8}{4}+\frac{3}{4}}$$

$$= 5 x^2 \sqrt[4]{x^3}$$

Convert a radical to
a rational exponent

$$\sqrt[10]{x^5} = x^{\frac{5}{10}} = \textcircled{x^{\frac{1}{2}}}$$

numerator is the power
denominator is the index

